M&T p. 190

Vector Analysis Identities

$A \times (B \times C) = \vec{B}(A \cdot C) - \vec{C}(A \cdot B)$

Jacobi P. 39 $\quad u \times (v \times w) + v \times (w \times u) + w \times (u \times v) = 0$

$\vec{\nabla}(fg) = f\,\vec{\nabla} g + g\,\vec{\nabla} f$

$\nabla(f/g) = \frac{1}{g^2}(g\nabla f - f\nabla g)$ Just quotent rule w/ ∇!

$V = (F \cdot \nabla) G$ has components $V^i = F \cdot (\nabla G^i)$

$\begin{cases} \nabla \cdot (F+G) = \nabla \cdot F + \nabla \cdot G \\ \nabla \times (F+G) = \nabla \times F + \nabla \times G \end{cases}$

$\nabla(F \cdot G) = (F \cdot \nabla) G + (G \cdot \nabla) F + F \times (\nabla \times G) + G \times (\nabla \times F)$

$\nabla \cdot (fF) = f\,\nabla \cdot F + F \cdot \nabla f$

$\quad = \nabla f \cdot F + f\,\nabla \cdot F$

$\nabla \cdot (F \times G) = G \cdot (\nabla \times F) - F \cdot (\nabla \times G)$

$\nabla \cdot (\nabla \times F) = 0 \qquad [\,d^2 = 0\,]$

$\nabla \times (fF) = f(\nabla \times F) + \nabla f \times F$

$\nabla \times (F \times G) = F(\nabla \cdot G) - G(\nabla \cdot F) + (G \cdot \nabla) F - (F \cdot \nabla)\vec{G}$

$\nabla \times (\nabla \times F) = \nabla(\nabla \cdot F) - \vec{\nabla}^2 F$

$\nabla \times (\nabla f) = 0$

$\nabla(F \cdot F) = 2(F \cdot \nabla) F + 2F \times (\nabla \times F)$

$\nabla^2(fg) = f\,\nabla^2 g + g\,\nabla^2 f + 2(\nabla f \cdot \nabla g)$

$\nabla \cdot (\nabla f \times \nabla g) = 0$ (follows from others)

$\nabla \cdot (f\nabla g - g\nabla f) = f\,\nabla^2 g - g\,\nabla^2 f$

$H \cdot (F \times G) = G \cdot (H \times F)$

$\quad = F \cdot (G \times H)$

$H \cdot [(F \times \nabla) \times G] = [(H \cdot \nabla) G] \cdot F - (H \cdot F)(\nabla \cdot G)$

Green's Ids p. 447

"int by parts" analogue $\quad \int_{\partial\Omega} f\,\nabla g \cdot n\,dS = \int_{\Omega} \left(f\nabla^2 g + \nabla f \cdot \nabla g\right) dV$

$\int_{\partial\Omega} (f\nabla g - g\nabla f) \cdot n\,dS = \int_{\Omega} \left(f\nabla^2 g - g\nabla^2 f\right)$

LCL inside cover

$\nabla_x\left(\frac{1}{\|x-y\|_2}\right) = \frac{-1}{\|x-y\|^2}(\hat{x} - \hat{y})$ what about ∇_y?

$\nabla\left(\frac{1}{\|x\|_2}\right) = \frac{-1}{\|x\|_2^2}\hat{x}$ (just $y=0$)

$1 + 2\left(\frac{\gamma}{J}\right) \wedge \gamma + \gamma$

Marsden & Tromba

Forms of Green's thm:

p.408
$$\int_{\partial D} P\,dx + Q\,dy = \int_{D} (Q_x - P_y)\,dx\,dy$$

p.411
$$\int_{\partial D} F\cdot ds = \int_{D} (\nabla\times F)\cdot \hat{e}_3\,dA$$

for $\vec{F} = \begin{bmatrix} P \\ Q \\ 0 \end{bmatrix}$

p.413
$$\int_{\partial D} \vec{F}\cdot\hat{n}\,ds = \int_{D} \mathrm{Div}(F)\,dA$$

Stokes p.422
$$\int_{S} \overrightarrow{(\nabla\times F)}\cdot\hat{n}\,dS = \oint_{\partial S} \vec{F}\cdot d\vec{s}$$

Divergence Thm
$$\int_{V} (\nabla\cdot F)\,dV = \int_{\partial V} \vec{F}\cdot\hat{n}\,dS$$