

Symbols

$Df_x, \delta f, Tf_x, f_{*x}, df_x, (\mathcal{L}_X f)(x)$

Frechet deriv

1)  $f: X \rightarrow Y$  nls  $Df_x$  is the unique linear map  $L$  such that

$$\lim_{\|h\| \rightarrow 0} \frac{\|f(x+h) - f(x) - Lh\|_Y}{\|h\|_X} = 0$$

2) Gateaux

$$\delta f(x; h) = \lim_{t \rightarrow 0} \frac{f(x+th) - f(x)}{t}$$

Note  $Df_x(h) = \delta f(x; h)$  for each fixed  $x, h \in X$ .

3)  $f: M \rightarrow N$  abstract mfd's

$$Tf: TM \rightarrow TN$$

$$[\varphi]_m \mapsto [f \circ \varphi]_{f(m)}$$

This map is identical with  $f_*$

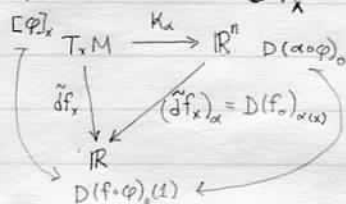
$$Tf_m: T_m M \rightarrow T_{f(m)} N$$

(4)  $f: M \rightarrow \mathbb{R}$

$df_x: T_x M \rightarrow \mathbb{R}$

L&S p. 391  
BOPP

Ignore the tilde ~  
It's just here because  
L&S use "d" too often



$$[\varphi]_x \mapsto (f \circ \varphi)'(0) = D(f \circ \varphi)_0(1)$$

$$(\tilde{df}_x)_\alpha (D(\alpha \circ \varphi)_0) = D(f \circ \varphi)_0$$

$$D(f \circ \alpha^{-1})_{f_x}$$

Lie Derivative

(5)  $f: M \rightarrow \mathbb{R}$

$$(\mathcal{L}_X f)(x) = (f \circ \varphi_x)'(0) = D(f \circ \varphi_x)_0$$

$X: M \rightarrow TM$

$\rightarrow$  inf gen of flow  $\varphi$

L&S  
p. 383-384