<u>Symbols</u> $Df_x$, $\delta f$, $Tf_x$, $f_{*x}$, $df_x$, $(\mathcal{L}_X f)_*(x)$

Frechet deriv

1) $f: X \to Y$ nls $\quad Df_x$ is the unique linear map $L$ such that

$$\lim_{\|h\| \to 0} \frac{\| f(x+h) - f(x) - Lh \|_Y}{\|h\|_X} = 0$$

2) Gateaux

$$\delta f(x;h) = \lim_{t \to 0} \frac{f(x+th) - f(x)}{t}$$

Note $Df_x(h) = \delta f(x;h)$ for each fixed $x, h \in X$.

3) $f: M \to N$ abstract mfds

$$Tf : TM \longrightarrow TN$$
$$[\varphi]_m \longmapsto [f \circ \varphi]_{f(m)}$$

This map is identical with $f_*$

$$Tf_m : T_m M \longrightarrow T_{f(m)} N$$

(4) $f: M \to \mathbb{R}$ $\quad df_x : T_x M \longrightarrow \mathbb{R}$

L&S p.391
BOPP

Ignore the tilde ~
It is just here because
L&S use "d" too often

$$\begin{tikzcd} {[\varphi]_x} & T_xM \arrow[r, "\kappa_\alpha"] \arrow[d, "\tilde{d}f_x"'] & \mathbb{R}^n \arrow[dl, "(\tilde{d}f_x)_\alpha = D(f_\circ)_{\alpha(x)}"] & D(\alpha \circ \varphi)_0 \\ & \mathbb{R} & & \\ & D(f \circ \varphi)_0(1) & & \end{tikzcd}$$

$$[\varphi]_x \longmapsto (f \circ \varphi)'(0) = D(f \circ \varphi)_0(1)$$

$$\underbrace{(\tilde{d}f_x)_\alpha}_{\underbrace{D(f \circ \alpha^{-1})}_{f_\alpha}} (D(\alpha \circ \varphi)_0) = D(f \circ \varphi)_0$$

Lie Derivative

(5) $f: M \to \mathbb{R}$ $\quad (\mathcal{L}_X f)(x) = (f \circ \varphi_x)'(0) = D(f \circ \varphi_x)_0$

L&S p.383-384 $\quad X: M \to TM$

↘ inf gen of flow $\varphi$