

$T: V \rightarrow V$ V has basis $\{e_1, \dots, e_n\}$

We want the matrix rep of T wrt basis $\{e\}$

Step 1 $T(e_i) = \{e_1, \dots, e_n\} \begin{bmatrix} a_i^{(1)} \\ \vdots \\ a_i^{(n)} \end{bmatrix} = \vec{A}_i$ Thus $T(\{e_1, \dots, e_n\}) = \{e_1, \dots, e_n\} [A]$

Step 2 so for $\vec{v} = \{e_1, \dots, e_n\} \begin{bmatrix} v^1 \\ \vdots \\ v^n \end{bmatrix}$

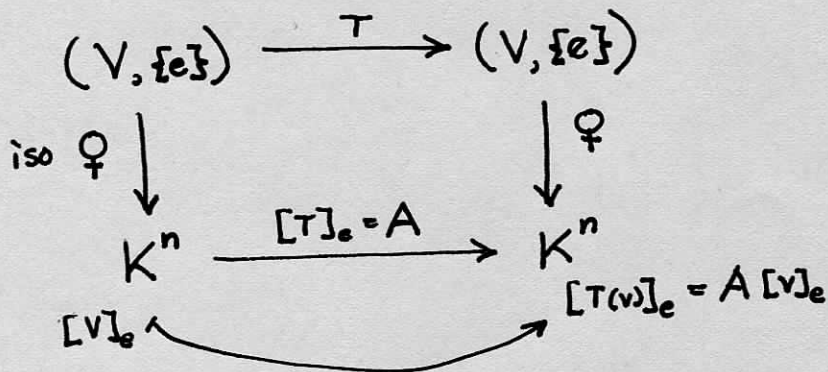
$$T(v) = T(\{e_1, \dots, e_n\} \begin{bmatrix} v^1 \\ \vdots \\ v^n \end{bmatrix}) = v^1 T(e_1) + v^2 T(e_2) + \dots + v^n T(e_n)$$

$$= v^1 \{e_1, \dots, e_n\} \vec{A}_1 + \dots + v^n \{e_1, \dots, e_n\} \vec{A}_n$$

$$= \{e_1, \dots, e_n\} (v^1 \vec{A}_1 + \dots + v^n \vec{A}_n)$$

$$= \{e_1, \dots, e_n\} [A] \begin{bmatrix} v^1 \\ \vdots \\ v^n \end{bmatrix}$$

This is $[Tv]_e = [T]_e [v]_e$



$$\begin{array}{ccc}
 \tilde{P}: \{ \text{Bases of } V \} & \longrightarrow & \{ \text{Bases of } V \} \\
 \{e\} & \longmapsto & \{e\} P = \{f\}
 \end{array}$$

let $v = \{f\} [v]_f$

then $v = \underbrace{\{e\} P}_{[v]_e} [v]_f \Rightarrow [v]_f = P^{-1} [v]_e$