Schuam's LA
P. 150-154

CHANGE OF BASIS

C.O.B. operator

$$\tilde{P}: \{\text{Bases for } V\} \longrightarrow \{\text{Bases for } V\}$$
$$[e] \longmapsto [e]P = [f]$$
(P: matrix)

Then in co-ords

$$[v]_f = P^{-1}[v]_e \quad \text{and} \quad P_{v_f} = v_e$$

Also, since $[f_i]_f = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \Rightarrow P_i = [f_i]_e$

$$\Rightarrow P = \begin{bmatrix} | & & | \\ [f_1]_e & \cdots & | \\ | & & | \end{bmatrix}$$

Change of Basis diagram

$$\begin{array}{ccc} (V,e) & \xrightarrow{\tilde{P}} & (V,f) \\ \downarrow \varphi & & \downarrow \varphi \\ K^n & \xrightarrow{P^{-1}} & K^n \\ [v]_e & & [v]_f = P^{-1}[v]_e \end{array}$$

Matrix representation of a Linear Operator

$$\begin{array}{ccc} (V,e) & \xrightarrow{T} & (V,e) \\ \downarrow & & \downarrow \\ K^n & \xrightarrow{[T]_e} & K^n \\ [v]_e & & [T(v)]_e \end{array}$$

Change of Basis affecting a linear map $T: V \to V$