9/24/2000

Velocity and Acceleration in Polar Co-ords

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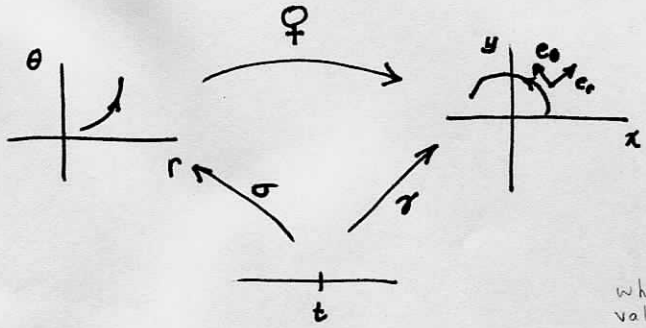
ch 1
p. 27-28



$$\text{♀}(r,\theta) = \begin{bmatrix} r\cos\theta \\ r\sin\theta \end{bmatrix} \begin{matrix} x \\ y \end{matrix}$$

when we multiply this out, it is just a vector-valued fcn, and we know how to differentiate that

$$\dot{x} = A\dot{p}$$

$$\gamma(t) = \text{♀}(\sigma(t))$$

$$\dot\gamma(t) = D\text{♀}_{\sigma(t)}(\dot\sigma(t)) \rightsquigarrow \begin{matrix} \dot x \\ \dot y \end{matrix} \begin{bmatrix} \dot\gamma_1 \\ \dot\gamma_2 \end{bmatrix} = \overbrace{\begin{bmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{bmatrix}\begin{bmatrix} \dot r \\ \dot\theta \end{bmatrix}}$$

$$= \dot r \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} + \dot\theta \begin{bmatrix} -r\sin\theta \\ r\cos\theta \end{bmatrix}$$

$$\dot{\vec\gamma} = \vec v = \boxed{\dot r\, \hat e_r + r\dot\theta\, \hat e_\theta}$$

we define the <u>unit</u> vectors

$$\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} \qquad \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

And for acceleration:

$$\ddot\gamma(t) = D^2\text{♀}_{\sigma(t)}(\dot\sigma, \dot\sigma) + D\text{♀}_{\sigma(t)}(\ddot\sigma)$$

$$\begin{bmatrix} [\dot r\ \dot\theta]\begin{bmatrix} 0 & -\sin \\ -\sin & -r\cos \end{bmatrix}\begin{bmatrix} \dot r \\ \dot\theta \end{bmatrix} \\ [\dot r\ \dot\theta]\begin{bmatrix} 0 & \cos\theta \\ \cos\theta & -r\sin\theta \end{bmatrix}\begin{bmatrix} \dot r \\ \dot\theta \end{bmatrix} \end{bmatrix} \longrightarrow \begin{bmatrix} -2\dot r\dot\theta\sin\theta - \dot\theta^2 r\cos\theta \\ 2\dot r\dot\theta\cos\theta - r\dot\theta^2\sin\theta \end{bmatrix}$$

Thus

$$\ddot\gamma(t) = 2\dot r\dot\theta \underbrace{\begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}}_{e_\theta} - r\dot\theta^2 \underbrace{\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}}_{e_r} + \ddot r \underbrace{\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}}_{e_r} + r\ddot\theta \underbrace{\begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}}_{e_\theta}$$

$$= (\ddot r - r\dot\theta^2)\hat e_r + (r\ddot\theta + 2\dot r\dot\theta)\hat e_\theta$$

$\square$

Help remembering the calculation:

$\dot x_1 = F^{①} = \sum_i (D_i h^1 \dot p_i)$ where I renamed "♀" as "h"

then we can compute $\frac{d}{dt}F^{①}$ by ordinary methods and factor out the $\dot p_i$, $\ddot p_j$

$$\ddot x_1 = \dot F^{①} = [\dot p_1\ \dot p_2]\begin{bmatrix} D_1^2 h^1 & D_2 D_1 h^1 \\ D_1 D_2 h^1 & D_2^2 h^1 \end{bmatrix}\begin{bmatrix} \dot p_1 \\ \dot p_2 \end{bmatrix} + [D_1 h^1\ \ D_2 h^1]\begin{bmatrix} \ddot p_1 \\ \ddot p_2 \end{bmatrix}$$

always $h^{①}$