For each pt x, there is a fcn f: $f:(x,\Omega) \to \vec{E}(x)$

• Bryan Roberts time reversal
• Yao B&A Mirror symm

8/25/2022

Rosen ASPFS

$E(x) = \int_\Omega$ (coulomb's law) ↓ basically

Symmetry

Start with $\mathcal{E}$ and work back to what Causes it (1)

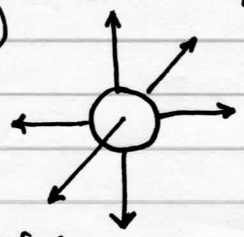
$\mathcal{E}$ = Electric field $\vec{E}$

p. 120 working thru the examples C = config of pos charge

Cause: ~~E field~~ unit sphere of charge (Spherically Symm)

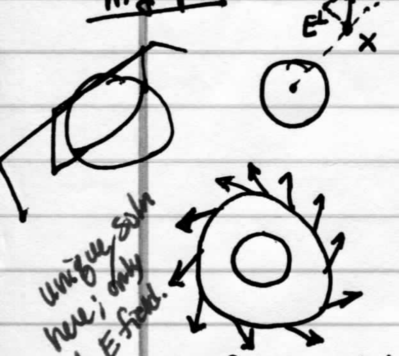
Symm(C) ⇒ Symm($\mathcal{E}$)

(ex 1)



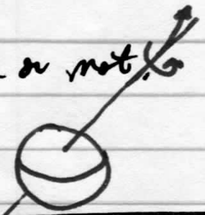
Effect: radial $\vec{E}$ field, spherically symm

my pf:



Suppose at some pt x the field was not radial; there was an $E^\perp_x$ component.
Reflect everything thru the radial ~~radial~~ plane thru x, ⊥ to $E^\perp_x$, then $E^\perp_x$ reflects to $-E^\perp_x$
BUT the charge sphere is identical after the reflection, so $\vec{E}(x)$ should be just as it was originally
we can't tell if reflection happened or not.

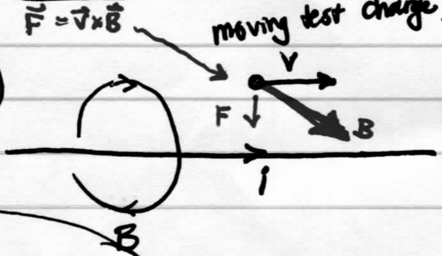
unique soln here; only 1 E field.



The field could be like this w/out violating rotational symm!
No Rotate around the radial line $\overline{OX}$.

moving test charge, going parallel with current in straight wire

(ex 2)



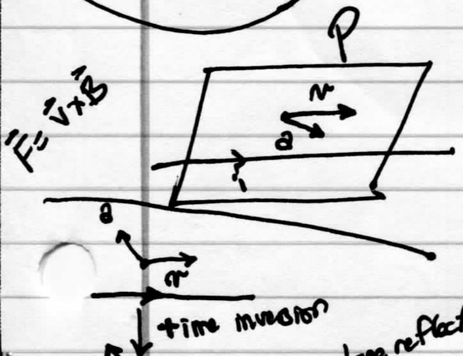
Find accel of test charge moving parallel to current carrying straight wire

Cause = current & moving test charge
Effect = accel of test charge

Tricky issues of how $\vec{B}$ field transforms under reflections

Symm group of Cause contains reflection thru plane of wire and charge P

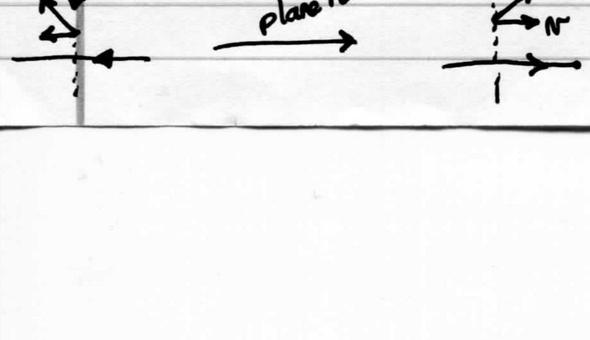
This must be a symm of the Effect
so accel can have no component outside the plane P



Symm Group also contains:
temporal inversion and reflection

$t \mapsto -t$
$x \mapsto -x$
$y \mapsto y$

This reverses the sense of all velocities

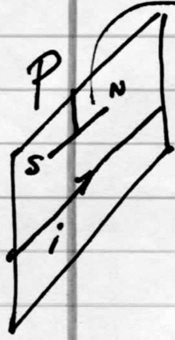


Arvoys Gyro

8/26/2022

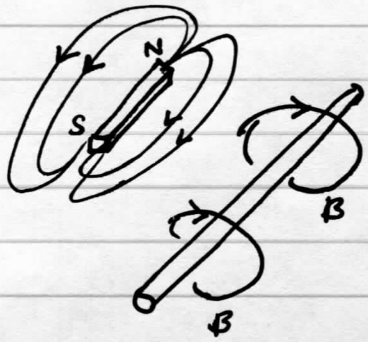
2

ex3 – From Symm Discovered book p. 122, 145 E. Machs paradox

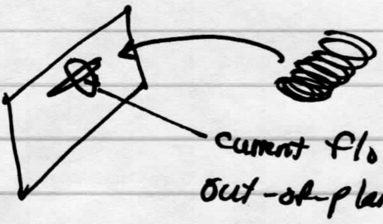


The needle deflects, apparently violating Symm!

But on closer examination



Bar mag is really a solenoid



current flow in the out-of-plane circle that is not preserved under reflection, so not symm.

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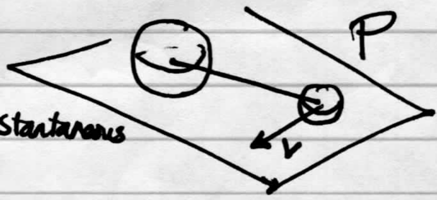
ex4 SD p. 110–112, ASPFS p. 121–123

The orbit of a planet around Sun lies completely in plane P and P runs thru centre of Sun

1. Sun & Planet spherically symm

what about Halo orbits bending out of plane?



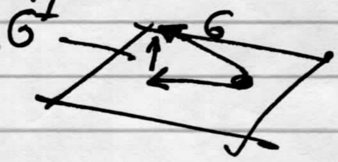
Cause = Sun & planet with instantaneous pos and velocity

Effect = accel of planet

My way Config is unchanged by reflection thru plane P

$G(x)$ = force on planet at pt x

Suppose there was an out-of-plane component $G^\perp$



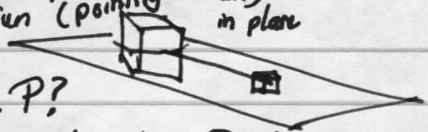
The book's arg seems to have a lot of problems, hypothesis missing.

we only use reflection symm, so any shape would work like this?

we must have (1) Planet's instantaneous $\vec{v}$ in plane

(2) straight line force from Sun (pointing to Sun) in plane

(3) $\vec{F} = m\vec{a}$



$m\ddot{\vec{\sigma}}(t) = \vec{F}(\sigma(t))$

Can the planet Q ever leave the plane P?

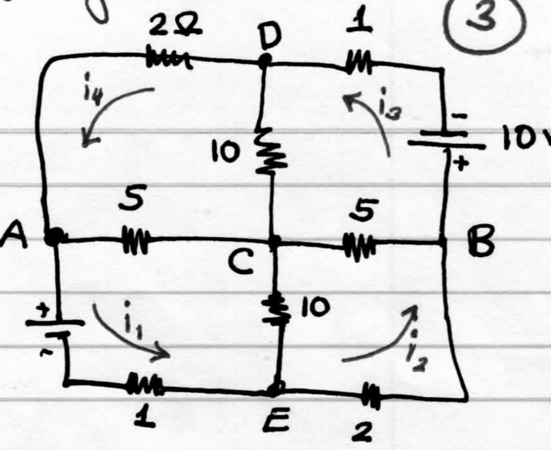
Equivalent to: can a curve σ leave the plane P if at any time t_0

we have $\sigma(t_0) \in P$, $\vec{\dot\sigma}(t_0) \in P$, $\vec{\ddot\sigma}(t_0) \in P$? and σ is determined by and initial conds.

A general curve could have $\vec{T}$ in plane, $\vec{N}$ in plane (they define a plane) and then twist out of the plane by torsion τ $\vec{B}$ right?

This is like Pierre Curies
original Crystalography

8/25/2002

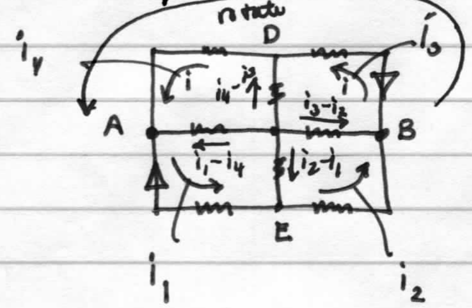


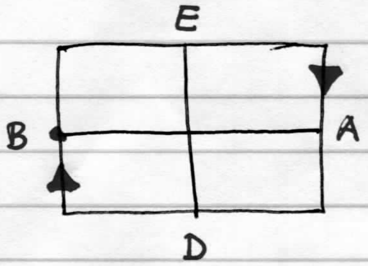
ex 5 SD p. 112-114

Cause = • EMF sources
• 8 resistors and connecting wires

Effect = • currents in each branch
• resulting voltages between all pts in circuit

arbitrarily assign i_1, i_2, i_3, i_4
and express other currents in terms of these



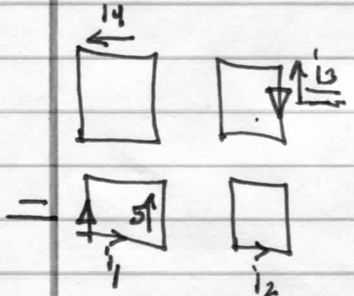


By comparison, we see $i_2' = i_4'$
$i_3' = i_1'$

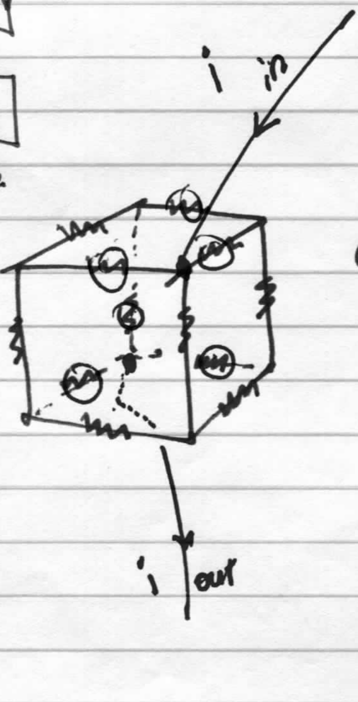
$V_{AC} = V_{BC}$
$V_{DE} = V_{ED}$

a priori $V_{DE} = -V_{ED}$

Thus $V_{DE} = 0$



ex 6



cube of resistors

$\mathcal{E}$ = fcn $\rho: S^2 \to \mathbb{R}$ giving charge density at each pt on the sphere.

8/20/2022

ASPFS

P. 128–129

This is an elaboration on the discussion in Shaw

ex 7 Charge distrib on a homog sphere

a priori, the charge does not have to be unif distrib over sphere.

Cause = charging device and the sphere
Effect = final charge distrib on sphere

How could we charge it in a symmetric way?

1. Deliver charge to S^2_q
2. Charge distribs itself by repulsion
3. Final configuration

ROSEN IS WRONG

another way to charge it is to touch it with another charged sphere

The cause has axial and reflection symm

rotations about line of wire — reflections thru all planes contain this line

NOT spherical symm — not all axes line

If we don't know the way the sphere was charged, the Effect is the set of all allowed distributions

No single charge distrib has to be spherically symm, Given 1 allowed distrib, any other distrib comes from a rotation, that is also allowed.

If the Cause is not suff to determine a uniq soln, then the Effect must be the set of all solutions.

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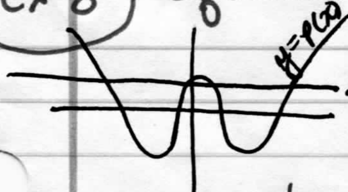
p131

ex 8 equations



Consider solns to polynomial eq $p(x) = r$ when p is only even powers of x, like

$$p(x) = ax^6 + bx^4 + cx^2 + d$$

let symm $S: x \mapsto -x$

written up

$$p(S(x)) = a(-x)^6 + b(-x)^4 + c(-x)^2 + d$$
$$= ax^6 + bx^4 + cx^2 + d = p(x)$$

If there is only ONE soln, then $Sx^* = x^*$ and only then x^* itself is symm!

Thus if x^* is a soln $p(x^*) = r$, then Sx^* is also a soln: $p(Sx^*) = r$ — S maps each soln to another soln.

$p \circ S = p$

9/28/2022

5a

Written up

youtube math symm

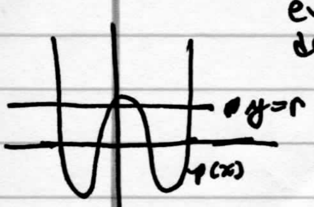
See also my other Sheets algebra problem

Revisiting the soln of polynomial eqs

even deg terms $p(x) = ax^6 + bx^4 + cx^2 + d$ $\qquad p(x) = r$

$S(x) = -x$ $\qquad$ we know $p(S(x)) = p(x)$

so if x^* is a soln, so is Sx^*



We can also re-cast this as

$F: \mathbb{R}^2 \longrightarrow \mathbb{R}$

$(x,y) \longmapsto p(x) - y$ $\qquad$ Then we seek to solve $F(x,y) = 0$

$F^{-1}(0) =$ ~~curve in the graph~~ set of discrete pts of intersection

if we fix $y = r$ then $F(x,r) = 0$ means $\quad p(x) - r = 0$ which is what we had before.

In this formulation,

$S = \begin{bmatrix} -1 & \\ & 1 \end{bmatrix}$ and horiz lines on graph are fixed by S

solns are where horiz line cuts fcn graph

NOW consider $q(x) = ax^5 + bx^3 + cx$ $\quad$ all odd degree terms

and $S(x) = -x$ $\qquad$ If we look at just $q: \mathbb{R} \to \mathbb{R}$

$q(S(x)) = -q(x) \Rightarrow$ then $\qquad S(q(x)) = q(S(x))$

$S \circ q = q \circ S$

$$\begin{tikzcd} \_ \arrow[r, "q"] & \_ \\ \_ \arrow[u, "S"] \arrow[r, "q"] & \_ \arrow[u, "S"'] \end{tikzcd}$$

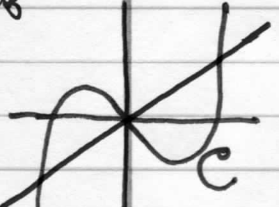
qualitative

But if we look at it this way our symm S only operates on the domain

$F: \mathbb{R}^2 \longrightarrow \mathbb{R}$

$(x,y) \longmapsto q(x) - y$

Written up



$F(x,y) = 0$ is what we seek

$S = \begin{bmatrix} -1 & \\ & -1 \end{bmatrix}$ $\qquad$ Then $F \circ S(x,y) = F(x,y)$ when restricted to C $= q(x) - y$

because $\quad q(-x) - (-y) = -q(x) - (-y) = 0$

$= q(x) - y = 0$

Symm is reflection thru origin; lines thru origin are fixed

when restricted to C $\quad F(x,y) = q(x) - y = 0$

$\Rightarrow y = q(x)$ $\qquad C = F^{-1}(0) =$ curve in graph

a line thru origin is $y = mx$

$F(x,y)\big|_C = q(x) - y = 0$

Thus we seek solns where this line intersects C

in this case only $F(S(x,y)) = -q(x) - (-y) \stackrel{?}{=} 0$

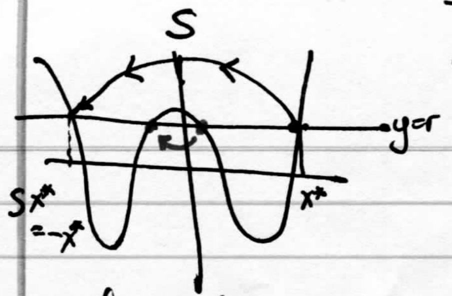
$\Rightarrow q(x) - y = 0$

$= F(x,y)$.

$q(x) = mx$ reduces order to an even poly $p(x) = m$ and we know that soln.

5/26/2022

S



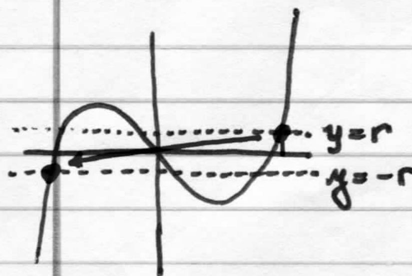
the solns are pts where line $y=r$ meets graph; both graph and line are preserved.

$S: \Sigma \to \Sigma$ set of solns

what about p all odd powers of x? Here it is a little more tricky

$p(x) = ax^5 + bx^3 + cx \qquad S(x) = -x \qquad p \circ S = S \circ p$

Here $p(S(x)) = -ax^5 - bx^3 - cx$
$= -p(x)$
$= S(p(x))$



$(x,y) \longmapsto (-x,-y)$

Written up

[redacted]

This has soln defined for all $t \in \mathbb{R}$

ex 9 Harmonic oscillator $\quad \ddot{y} + a^2 y = 0 \quad$ where a is const.

This is invariant under $\quad t \longmapsto t+b$
$t \longmapsto -t$
$y \longmapsto cy \quad$ and $\quad y \longmapsto -y$ really covered by prev case.

(a) $\mathbb{R}$ or no translations!

If $\varphi : [a,b] \to \mathbb{R}$ is a soln

$\varphi \in$ 2 times difb fcns $= C^{1+d}(I \to \mathbb{R}) =: \cancel{C}\ D$ call it D since C mean "Cauchy"

Time Translation Then $S(\varphi)$ should also be a soln

Let $S_b : C \longrightarrow C$

$L(S\varphi)$ is a soln $\qquad \varphi \longmapsto \varphi \circ \tau_b$ where $\tau_b : t \longmapsto t+b$

$L(\cdot) = \left(\frac{d^2}{dt^2} + aI\right)(\cdot)$

so $\varphi(\tau_b(t)) = \varphi(t+b)$ on $(\varphi \circ \tau_b)(t)$

then $(\varphi \circ \tau)'' + a^2(\varphi \circ \tau)$

$\tau : [a-b, 0] \longrightarrow [a,b]$

[redacted] But if this doesn't matter

L: $\qquad \varphi'(\tau)\tau' = \varphi'(t+b)\,1$

$\varphi''(t+b) + a^2\varphi(t+b) = 0$

(b) Time reversal Lets consider $S_n : C \longrightarrow C$
$\varphi \longmapsto \varphi \circ n \quad$ where $n(t) = -t$

Then $L(S_n\varphi)_{(t)} = (\varphi \circ n)'' + a^2(\varphi \circ n) = \varphi''(-t) + a^2\varphi(-t) = 0$

again because domain is $t \in (-\infty, \infty)$ so $-t$ makes no difference.

scalar mult

Written up

Harmonic oscillator, cont'd.

Ⓒ Let $S_c : C \longrightarrow C$

$\varphi \longmapsto c\varphi$ where c is any pos or neg number

then $L(S_c\varphi)_{(t)} = (c\varphi)''(t) + a^2(c\varphi)(t)$

$= c\varphi''(t) + a^2 c\varphi(t) = c(\varphi''(t) + a^2\varphi(t))$

$= c(0) = 0$ ✓

What Rosen does is give the explicit soln:

$y = A\sin(at + B)$ and apply each of these transforms explicitly to it.

$y' = A\cos(at+B)\cdot a$

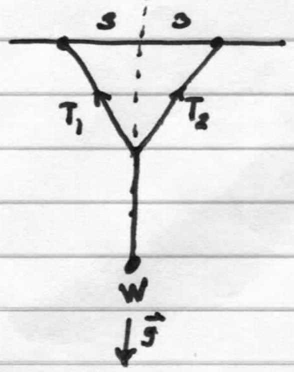
$y'' = -A\sin(at+B)\cdot a^2$

then $y'' + a^2 y = -a^2 A\sin(at+B) + a^2 A\sin(at+B) = 0$

(ex) Here is another type of prob: Statics

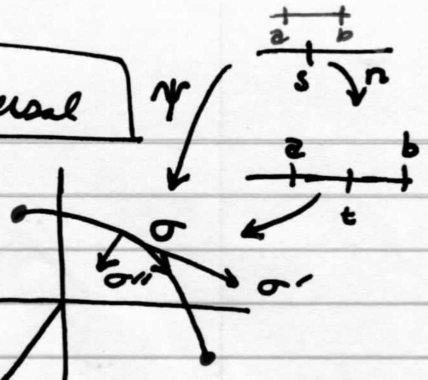
tension $T_1 = T_2$ by reflection symm

$\vec{g}$ is a polar vector, so it reflects nicely.



9/4/2022

$\boxed{\text{Newton's Laws admit time reversal}}$



we want to show $m\ddot{x} = F(x)$ is invariant under time reversal

Let $m\ddot{\vec{\sigma}}(t) = F(\sigma(t))$ on $[a,b]$

σ is a soln $\quad \sigma(a) = x_a$

$\sigma'(a) = \vec{V}_a$

$\sigma''(a) =$

what about tracing the path in the opposite direction?

any Newtonian system (say many particles or RBs) can be reversed mathematically. If $\{\sigma_i\}_{i=1}^N$ is a set of traj following from Newton's Laws then $\{\tilde{\sigma}_i\}_{i=1}^N$ is also a soln. If we run the film backwards, we can't tell if we are running in forwards or backwards

Let $\Psi(s) := \sigma(n(s))$

Then

$$\Psi'(s) = \sigma'(n(s))\,n'(s) = \sigma'((a+b)-s)\cdot -1 = -\sigma'(\cdot)$$

where $n(s) = (a+b)-s$ — if $[-b,b]$ then $-s$

$n(b) = a$

$n(a) = b$

$$\Psi''(s) = +\sigma''((a+b)-s)$$

what if interval was $[-b,b]$? then $a+b = -b+b = 0$

Since $a+b-s \in [a,b]$ we know $F(\sigma(a+b-s))$ is well defined.

Thus $m\sigma''(a+b-s) = F(\sigma(a+b-s))$

That is

$$\boxed{m\Psi''(s) = F(\Psi(s))}$$

The reversed path is a soln

How do we justify writing it as $m\sigma''(-t) = F(\sigma(-t))$?

Fix any pt q in $[a,b]$

$q = a+t$ and $q = b-s$

$\sigma(q) = x_q$

$\sigma'(q) = \vec{v}_q$

that same pt q is specified by!

$q = (a+b)-s$

$\Rightarrow s = (a+b)-q$

$\Psi(s_q) = \sigma(q) = x_q$ ✓

$\Psi'(s_q) = -\sigma'((a+b)-s_q)$

$= -\sigma'(q) = -v_q$ □