p.170

Gamma distrib $\Gamma(\lambda,\alpha)$ $\lambda>0$, $\alpha>0$

See also Rice MSADA p.49-50

pdf
$$g_{\lambda\alpha}(x) = \begin{cases} \dfrac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x} & x \geq 0 \\ 0 & x<0 \end{cases}$$

Ross likes to write
$$\frac{\lambda e^{-\lambda x}(\lambda x)^{\alpha-1}}{\Gamma(\alpha)}$$

Presumably to emphasize it is $\mathcal{E}$ distrib when $\alpha=1$.

rate (or $1/\lambda$ scale) param

Shape param

$$\frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}$$

The Gamma family of distribs are a flexible class that includes $\mathcal{E}$xp, $\chi^2(n)$ and n-Erlang.
$\Gamma(n,\lambda)$ models the total time for n unpredictable events (like earthquakes) to occur.
n-Erlang

$\Gamma(t)$ is the Gamma fcn
$$\boxed{\Gamma(t) := \int_0^\infty e^{-y} y^{t-1}\,dy}$$
for $t>0$

[Marsden ℂ Analysis: t can also be Complex, but we won't go there]

There is a vast literature on Γ's many forms and properties. It generalizes factorials to non-integers (pos).

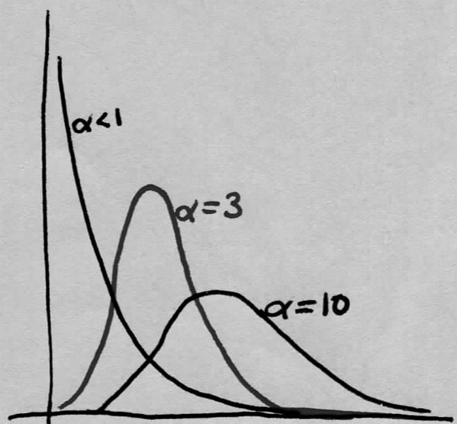
Lemma $\Gamma(1) = 1$

Pf. $\Gamma(1) = \int_0^\infty e^{-y}\,1\,dy = -e^{-y}\Big]_0^\infty = -[0-1] = +1$

Lemma $\Gamma(t) = (t-1)\Gamma(t-1)$ Here I think we need $t>1$

Pf. $$\int_0^\infty e^{-y} y^{t-1}\,dy \overset{\text{parts}}{=} \underbrace{-e^{-y}y^{t-1}\Big]_{y=0}^\infty}_{-[0\cdot\infty - 1\cdot 0] = 0} - \int_0^\infty -e^{-y}(t-1)y^{(t-1)-1}\,dy$$
$$+ (t-1)\int_0^\infty e^{-y} y^{(t-1)-1}\,dy$$
$$(t-1)\,\Gamma(t-1) \quad \square$$

$u = y^{t-1}$
$dv = e^{-y}dy$
$du = (t-1)y^{t-2}$
$v = -e^{-y}$



Lemma For any pos integer $n>1$ $\Gamma(n) = (n-1)!$

Pf. $\Gamma(n) = (n-1)\Gamma(n-1) = \ldots = (n-1)!\,\Gamma(1) = (n-1)!$

Lemma $\Gamma(1/2) = \sqrt{\pi}$ w. Elderton FCAC p.153, p.91

Pf. Spivak COM p.74 gives us $\int_{-\infty}^\infty e^{-x^2}dx = \sqrt{\pi} \overset{\text{Symm}}{\Longrightarrow} \sqrt{\pi} = 2\int_0^\infty e^{-x^2}dx$

let $y = x^2$
then $dy = 2x\,dx$
or $2dx = \frac{1}{x}dy = y^{-1/2}dy$

$$= \int_{y=0}^{\infty^2=\infty} e^{-y} y^{-1/2}\,dy$$
$$= \int_0^\infty e^{-y} y^{(1/2-1)}\,dy = \Gamma(1/2) \quad \square$$

More in my ch 6 writeup

$\Gamma(5/2) = \frac{3}{2}\Gamma(3/2) = \frac{3}{2}\cdot\frac{1}{2}\Gamma(1/2) = \frac{3}{4}\sqrt{\pi}$

Tennenbaum & Pollard ODE has nice discussion of Γ properties and Laplace transform relationship.