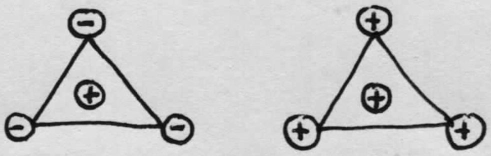
Feynman ch 5

Ch 5.2 EQ in electrostatic field

Earnshaw's Thm a charged particle cannot be held in Stable EQ by electroStatic forces alone.



In the planar cases shown, the center charge is stationary – but would move out under the tiniest perturbation.

Pf. Consider a pt x, we want to put a pos charge q there. If x would be a stable EQ position, there would be an E field that would restore a particle to position x. Imagine a Gaussian Surf S

The field points inward $\Rightarrow$ neg flux

Gauss $\displaystyle\int_S E\cdot n\,dA = \frac{1}{\epsilon_0} Q$ The LHS is a neg number, but RHS = 0 since no charge enclosed (especially no neg charge)

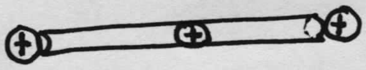
So there can be no pt x where all directions have E pointing inward.

▷ Feynman purports to say this is true for larger objects as well, like a charged rod.

other sources claim the same idea for magnetostatic fields — we need to have dynamic fields to levitate a magnet, for example.

The arg given above (due to Maxwell) is also said to not hold up under mathematical criticism. Another so-called flawed arg uses φ as a harmonic fcn and not having a min at an interior pt, so not stable EQ.

▷ Feynman also shows



The charge in the center of tube is in Stable EQ due to mechanical constraint of the tube (charges at ends must be held in place).

▷ **Ch 5.3 EQ with Conductors**

Charged conductors also cannot hold a charge in stable static EQ, by some potential energy arg.

▷ **ch 5.4 Stability of Atoms** – Definitely not electrostatically stable, only by quantum mech.

▷ Now let's use Gauss' Law to find E for certain Sym distributions of charge.

Gauss' Law is always true, but not always helpful to solve problems.

It can help us when we have spherical, cylindrical, or planar sym.

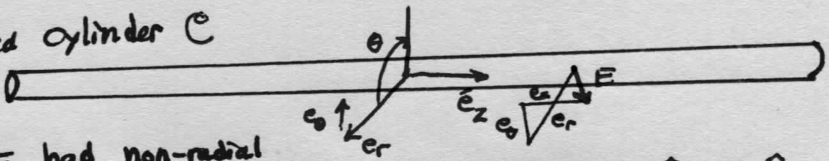
(we can also have a combination of these types of objects, and use superposition)

2 co-ord views: • The pt x must be the same. 4/26/2025

• The body C and Environment must look identical to infinite magnification

Ch 5.5 The field of a line of charge

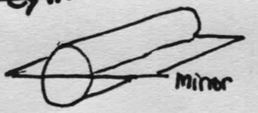
Consider a very long, uniformly charged cylinder C



claim: By sym $\boxed{\vec{E} = E(r)\,\hat{e}_r}$

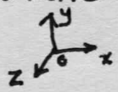
choose some pt x and Suppose E had non-radial components $\longrightarrow$ $E = E_1\hat{e}_r + E_2\hat{e}_\theta + E_3\hat{e}_z$ at pt x.

~~choose cylindrical co-ords~~



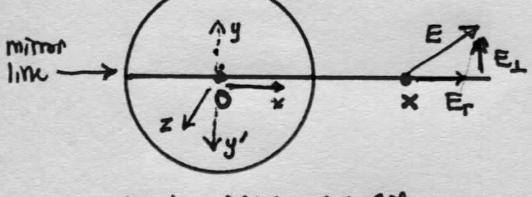
The cylinder is sym wrt a horiz plane thru centerline. reflection sym

Observer O_1 has co-ord axes



O_2 has mirror image

reflection fixes origin, x, z axes and the pt x



End view – we are inside C, looking lengthwise

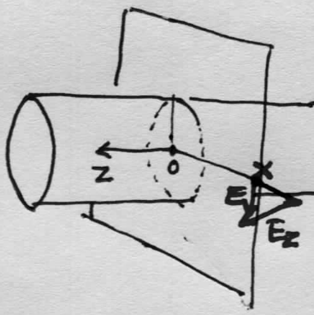
The cylinder looks identical to O_1 and O_2. At any specific co-ord, say $\langle 1,2,3\rangle$, O_1 and O_2 see something identical, but O_2 is seeing mirror image of the pt O_1 sees (unless pt is the the plane, then they are the same. So x is the same pt for both).

Now consider $\vec{E}(x)$. x is the same physical pt, AND has same co-ords to O_1 and O_2.

But if $E(x)$ had a y component $E_\perp$, then O_1 would see $+E_\perp$ in his axes, but O_2 would see $-E_\perp$ in his axes. Both viewpoints (co-ord choices) are equally valid

["You are looking at the mirror reflection". "No, you are looking at reflection"]

Thus $E_\perp$ must be 0.

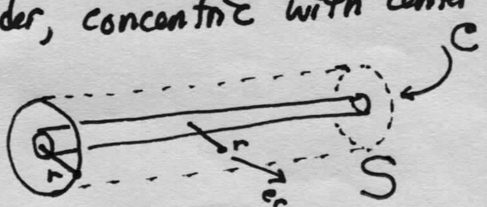


Likewise if E(x) had a nonzero $\hat{z}$ component. Reflect thru a $\perp$ mirror. Here we need C to be infinitely long, ~~or we need to be "near" the midpt.~~

$\Rightarrow \boxed{\vec{E}(x) = E(r)\,\hat{e}_r}$ in cylindrical co-ords

▷ Let the Gaussian surf S be another cylinder, concentric with center line.

$$\int_S \vec{E}\cdot\hat{n}\,dA = \frac{1}{\epsilon_0} Q_{inside}$$



Length L, we can let $L \nearrow \infty$

$$\int_0^L \int_0^{2\pi} E(r)\, r\, d\theta\, dz = \frac{1}{\epsilon_0}\int_0^L \lambda\, dz$$

First, let C just have linear charge density λ (Feynman's line of charge)

$$E(r)\, r\, 2\pi \cancel{L} = \frac{\cancel{L}\lambda}{\epsilon_0}$$

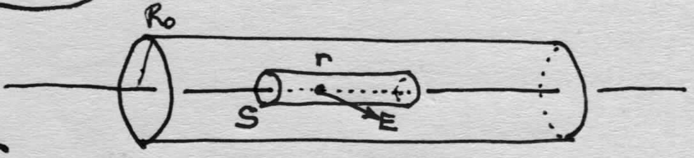
$$\vec{E}(r) = \frac{\lambda}{2\pi\epsilon_0}\frac{1}{r}\hat{e}_r$$ No need to let $L \to \infty$

Cont'd →

4/27/2025

3

Now consider Griffiths long cylinder p. 72 ex 2.3



charge density $\rho = kr$ for $0 < r < R_0$
radial dependence

what is the E field <u>inside</u> at a dist r from center line? Again we have sym that gives $\vec{E} = E(r)\hat{e}_r$

$$\int_S E\cdot n\,dA = \frac{1}{\epsilon_0} Q_{inside}$$

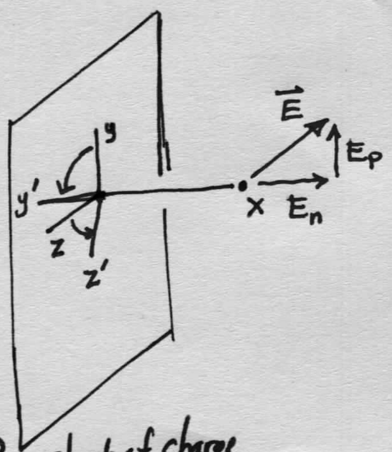
just like prev page

$$E(r)\, \cancel{2\pi} \cancel{r} \cancel{L} = \frac{1}{\epsilon_0}\int_0^L\int_{u=0}^r\int_0^{2\pi}(ku)(u\,d\theta)\,du\,dz = \frac{\cancel{2\pi} k \cancel{L} r^{\cancel{3}\,2}}{3\epsilon_0}$$

$$\vec{E}(r) = \frac{k}{3\epsilon_0} r^2 \hat{e}_r \quad \text{for } 0 < r < R_0$$

▷ <u>a sheet of charge; 2 sheets</u>

First, Sym arg to narrow down the possibilities for E



P = sheet of charge.

Claim $E = E(x)\hat{e}_x$ where $e_x = \hat{n}$ normal to plane

Choose co-ord sys where X is on x axis any y, z axes are in the plane P

P is invariant under rotations and reflections.

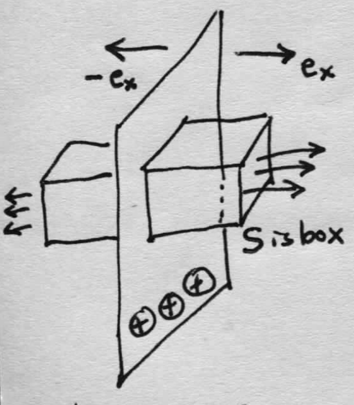
We could do the same reflection arg as before, but lets do a rotation

Observer O_1 has axes so parallel component E_p is in y dir.
Rotate 90° about x axis. X stays fixed, P is identical but O_2 would see E_p along z axis

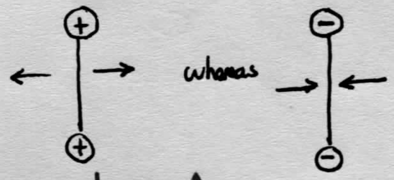
$\Rightarrow\Leftarrow$ $\quad E_p \stackrel{!}{=} 0$ and $\vec{E}$ is normal to plane P.

By translation sym, this holds for any X.

▷ Choose Gaussian box S as shown. Only front and back faces get any flux
Charge density σ per unit area. Box faces each have area A



Gauss $\displaystyle\int_S E\cdot n\,dA = \frac{1}{\epsilon_0} Q$



$$\int_{S_{front}} E\hat{e}_x\cdot\hat{e}_x\,dA + \int_{S_{Back}} E(-\hat{e}_x)\cdot(-\hat{e}_x)\,dA = \frac{1}{\epsilon_0}\sigma A$$

dist, abs value

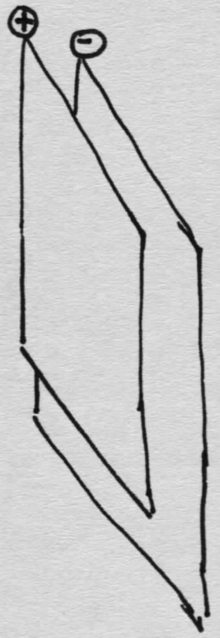
$$E(x)A + E(x)A = \frac{1}{\epsilon_0}\sigma A \Rightarrow \boxed{E(x) = \frac{\sigma}{2\epsilon_0} \quad \text{No dependence on dist } x}$$

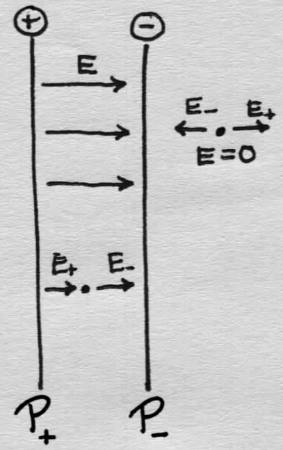
Pts in both directions ← →

Invariance of Scale Sym

Feynman ch 5 4/27/2025

▷ Now consider 2 parallel planes P_+ and P_-. They are oppositely charged.





From prev problem, and superposition E is normal to planes, and const wrt distance

Thus for a pt x outside both planes

$$\vec{E}(x) = -\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = 0$$

In between

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

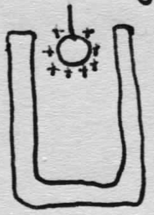
pos charge repelled by P_+ and attracted by P_-

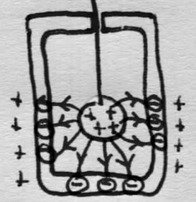
Ch 5.8 Is the field of pt charge exactly $1/r^2$?

First lets give Faraday's Ice Pail experiment to show all static charge is on the outside of a conductor [conductors will be discussed in detail in next section] This follows from Gauss' Law and Gauss depends on $1/r^2$ (Gauss correct ⇒ $1/r^2$ correct) (I think logically the ch 5.9 discussion should have come first – see that for any questions) We will also discuss special properties of a <u>spherical</u> shell.

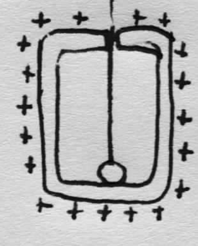
[Halliday & Resnick p. 453]

With insulated thread, Lower a charged ~~metal~~ ball into a metal can





Put on lid so "inside" is clearly defined



once the ball touches, the charge [...] off. Only the <u>outside</u> of [...] has charge that can be pi[...] by another ball. A ball put in[...] up nothing. E = 0 in cav[...]

as we shall see, Gauss predicts E = 0 since no charge inside we can measure this and thus confirm Gauss.

If the exponent of r were anything other than −2, Gauss would not give E = 0

$$E = \frac{kQq}{r^2}\hat{e}_r$$

[Could any other fcn of r do it? Not simple powers but something totally different?]

Maxwell vol 1 p. 36 talks about this

Let's give more details on next page →

Feynman ch 5.9

E Fields inside a Conductor

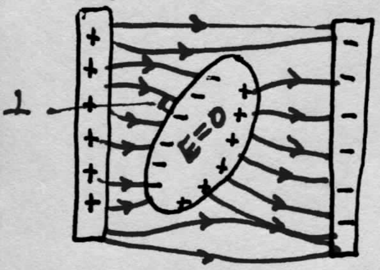
Griffith p. 88-90
Purcell p. 91-95

For our purposes, we are taking a Conductor C to be a solid body (usually metal) that has an "electron sea" where e^- can flow around freely, but they can't jump from the surface to leave C (this would require very high voltage to make a spark discharge)

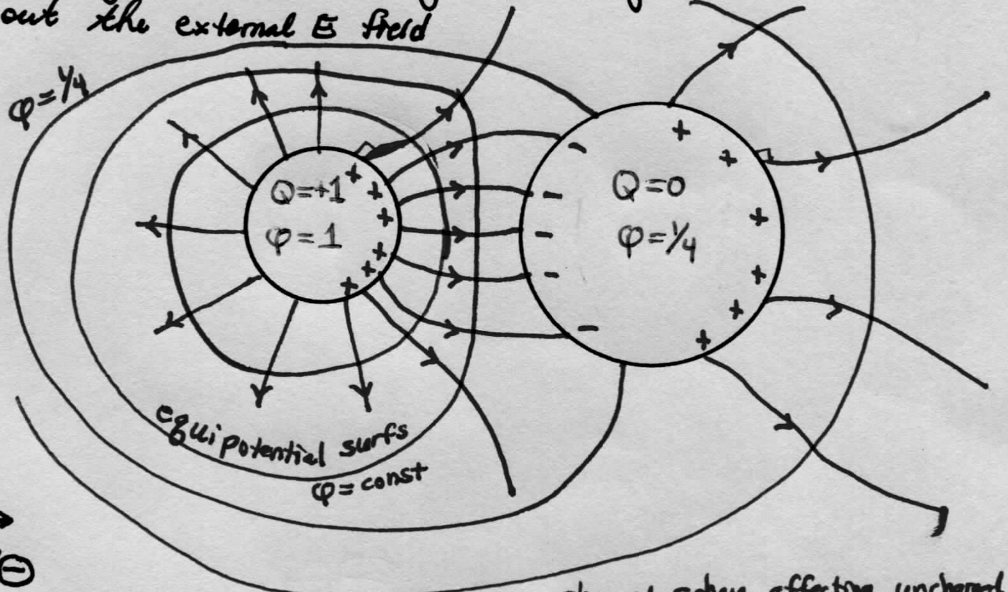
There are 3 sections (I) The conductor in an E field
(II) Local field at surf of C (this is on sheet 10)
(III) Cavities in C and electrostatic shielding

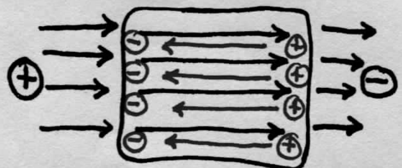
(I) ① E=0 inside the metal of a conductor - This is a consequence of Conservation of Energy. Since e^- can flow freely, they do so until they have arranged themselves in a way that cancels out the external E field

Purcell p.91



External E field is distorted by C (so my idea that the external field is not changed is wrong)





Here we can see the metal does not "block" the external E, but instead it is canceled out inside by superposition

Purcell p.94 Charged sphere affecting uncharged sphere. (Some errors in drawing - E lines should be ⊥ at surf)

How do we know the e^- come to a static state and don't keep moving around? Flowing e^- cause heating (Joule heating) and that is not observed. Also, stationary charges outside making an E field do not radiate any "thing"; they don't lose energy. So no work can be done inside the metal.

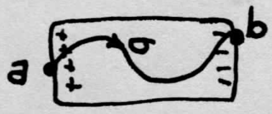
② Charge density $\rho \equiv 0$ inside C (equal pos & neg)

By Gauss $(\nabla \cdot E)(x) = \frac{1}{\epsilon_0}\rho(x)$ Since $E \equiv 0$, $\nabla \cdot E = 0 \Rightarrow \rho(x) = 0 \ \forall x$ inside
OR For any Gaussian surf wholly inside, no E flux $\Rightarrow$ no charge inside surf

③ The only place left for the excess charge to be is on the Surface of C (More details on this coming up, hinting that all charge on surf is the lowest energy configuration).

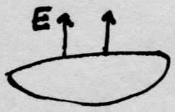
(4) C is an equipotential body — even at pts on the surf

Integrate $-\int_\sigma E\cdot ds = \underbrace{-\int_a^b E\cdot ds}_{\varphi(b)-\varphi(a)} = \int_a^b \vec{0}\cdot ds = 0$



$\Rightarrow \varphi(b) = \varphi(a)$ for any $a, b \in C$ even if on ∂C

(5) at the surf $E \perp C$ locally



This must be true or else charges would flow along the surf due to tangential component.

This leads us to section (II) but first I want to make a detour and discuss the electrostatic energy of a charge configuration Feynman ch 8

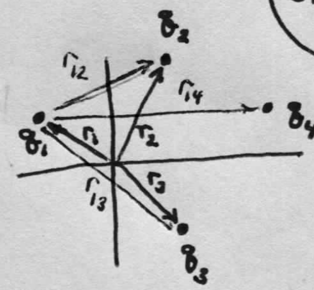
▷ what is the total work to assemble a collection of charges near origin? (or any pt for that matter)

Griffith p. 92

It takes 0 work to move in the first charge q_1

Now bring in q_2 $\quad \frac{W}{q_2} = -\int_\infty^r E\cdot ds = \varphi(r) = \frac{q_1}{4\pi\epsilon_0}\frac{1}{r}$

[sheet (3) from ch 4]



$\Rightarrow W_2 = k\frac{q_1 q_2}{r} = k\frac{q_1 q_2}{r_{12}} =$

Bring in q_3

$W_3 = \frac{k q_3 q_1}{r_{13}} + \frac{k q_3 q_2}{r_{23}}$

$W_4 = \frac{k q_4 q_1}{r_{14}} + \frac{k q_4 q_2}{r_{24}} + \frac{k q_4 q_3}{r_{34}}$

The total work $W = k\left(\underbrace{\frac{q_1 q_2}{r_{12}}}_{W_2} + \frac{q_1 q_3}{r_{13}} + \frac{q_1 q_4}{r_{14}} + \frac{q_2 q_3}{r_{23}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}\right)$ $\quad W_3 \quad W_4$

$\Rightarrow W = k\sum_{i=1}^{n}\sum_{\substack{j=1\\ j>i}}^{n}\frac{q_i q_j}{r_{ij}}$ $\rightsquigarrow$ or we can count each pair twice and divide by 2

$W = \frac{1}{2}\sum_{i=1}^{n} q_i \left[\overbrace{k\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{q_j}{\|r_j - r_i\|}}\right]$ This is $\varphi(x_i)$ (or $\varphi(r_i)$ in that notation)

This has permutation sym — it doesn't matter in which order the charges are brought in.

What about for a solid body Ω?
we'd just transform that formula as

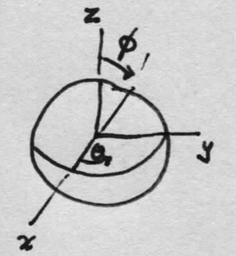
$$W = \frac{1}{2}\int_{x\in\Omega}\rho(x)\Big[\underbrace{\int_{\substack{y\in\Omega\\ y\neq x}}\frac{k\rho(y)\,d^3y}{\|y-x\|}}_{\varphi_\Omega(x)}\Big]d^3x = \frac{1}{2}\int_\Omega \rho\,\varphi_\Omega\,dV$$

This is Griffith eq (2.43) in my notation

but in practice, it is a little different

Let's do some examples →

We are going to calculate the energy of a sphere $S^2(0,R)$ and a unif ball $B^3(0,R)$. Then we see the charge all on the surf is a lower energy config. Then make the bold claim that this is true for any shape!



ex 2.8 Griffith p. 94 Unif charge Q over spherical shell $S^2(0,R)$

$$W = \frac{1}{2}\int_S \rho\varphi\, dV \text{ becomes } \frac{1}{2}\int_S \sigma \varphi_S\, dA \quad \text{and} \quad \sigma = \frac{Q}{\text{area}} = \frac{Q}{4\pi R^2}$$

$$= \frac{1}{2}\int_{\phi=0}^{\pi}\int_0^{2\pi}\left(\frac{Q}{4\pi \cancel{R^2}}\right)\left(\frac{KQ}{R}\right)\cancel{R^2}\sin\phi\, d\theta\, d\phi = \frac{1}{2}\frac{KQ^2}{4\pi R}\int_{\phi=0}^{\pi}\int_0^{2\pi}\sin\phi\, d\theta\, d\phi$$

$$= \frac{1}{2}\frac{KQ^2}{\cancel{4\pi} R}\cancel{4\pi} = \boxed{\frac{1}{2}\frac{KQ^2}{R}}$$

Prob 2.32 and 2.33

First let's do it Feynman's way, where we build the ball layer upon layer

$W = \frac{KQq}{r}$ we know from ch 4 sheet 3

For unif ball of total charge Q $\rho = \frac{Q}{\frac{4\pi}{3}R^3}$

$$dW = K\frac{Q}{r}dq \qquad Q(r) = \rho\frac{4}{3}\pi r^3 \qquad \underset{dQ}{} \qquad dQ = \rho 4\pi r^2 dr$$

$$= K\frac{\left(\rho\frac{4}{3}\pi r^3\right)}{r}\left(\rho 4\pi r^2 dr\right)$$

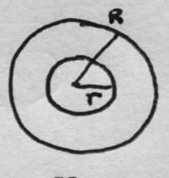
$$W = \int_0^R K\rho^2\frac{(4\pi)^2}{3}r^4 dr = K\rho^2\frac{(4\pi)^2}{3}\frac{1}{5}R^5 = \frac{K}{5}\left(\rho\frac{4\pi}{3}R^3\right)\left(\rho\frac{4\pi}{3}R^3\right)\frac{3}{R}$$

$$= \boxed{\frac{3}{5}\frac{KQ^2}{R}}$$

▷ Now let's do it Griffith's way $W = \frac{1}{2}\int_{B(0,R)} \rho\varphi_B\, dV$

NOTE: Here we are NOT building the ball by layers; we must integrate over a special φ on the inside. We found this on ch 4 sheet 8

$$W = \frac{1}{2}\int_{r=0}^{R}\int_{\theta=0}^{\pi}\int_0^{2\pi}\overbrace{\rho(r,\theta,\phi)}^{\text{no dependence}}\varphi_B(r,\overbrace{\theta,\phi}^{})\, r^2\sin\phi\, d\theta\, d\phi\, dr$$



$$= \frac{1}{2}\rho\int_0^R\frac{KQ}{2R}\left(3-\frac{r^2}{R^2}\right)r^2\overbrace{\left[\int_0^{\pi}\int_0^{2\pi}\sin\phi\, d\theta\, d\phi\right]}^{4\pi}dr \qquad \varphi_B(r) = \frac{KQ}{2R}\left(3-\frac{r^2}{R^2}\right)$$

$$1 - \frac{1}{5}$$

$$= \frac{\rho KQ\, 4\pi}{2\cdot 2R}\int_0^R\left(3r^2-\frac{1}{R^2}r^4\right)dr = \frac{\rho KQ\, 4\pi}{4R}\left[R^3 - \frac{1}{\cancel{R^2}}\frac{R^{\cancel{5}\,3}}{5}\right] = \frac{\rho KQ\, 4\pi}{\cancel{4}R}\left[\frac{\cancel{4}}{5}R^3\right]$$

$$= \frac{KQ}{5R}\left(\rho\frac{4}{3}\pi R^3\right)3 = \boxed{\frac{3}{5}\frac{KQ^2}{R}} \quad \text{Agrees with Feynman}$$

Sphere $= \frac{1}{2}\frac{KQ^2}{R}$ Solid ball $= \frac{3}{5}\frac{KQ^2}{R}$ $\frac{1}{2} < \frac{3}{5}$ So sphere has a little less energy while we are on this digression about electrostatic energy, let's give another way to express it: $W = \frac{1}{2}\int_\Omega \rho\varphi\, dV$ Re-write in terms of $\vec{E}$.

Griffith §2.4.3 p. 93
Energy of Cont charge Distrib
Shadowitz TEF p. 173-176

Vector identity: $\nabla\cdot(f\vec{G}) = \nabla f\cdot\vec{G} + f(\nabla\cdot\vec{G})$

pf: $D_x(fG^1) + D_y(fG^2) + D_z(fG^3) = f_x G^1 + fG^1_x + f_y G^2 + fG^2_y + f_z G^3 + fG^3_z$
$= f_x G^1 + f_y G^2 + f_z G^3 + fG^1_x + fG^2_y + fG^3_z$
$= \nabla f\cdot G + f(\nabla\cdot G)$ □

$W = \frac{1}{2}\int_\Omega \rho\varphi\, dV$ but $\rho = \epsilon_0(\nabla\cdot E)$ by Gauss $\Rightarrow \frac{\epsilon_0}{2}\int_\Omega \underbrace{(\nabla\cdot E)\varphi}\, dV$

By the identity $\nabla\cdot(\varphi E) = \nabla\varphi\cdot E + \underbrace{\varphi(\nabla\cdot E)}$

$$= \frac{\epsilon_0}{2}\left[\int_\Omega \nabla\cdot(\varphi E)\, dV - \int_\Omega \nabla\varphi\cdot E\, dV\right]$$

Div Thm

$$= \frac{\epsilon_0}{2}\left[\int_{\partial\Omega}\varphi E\cdot n\, dA - \int_\Omega \underbrace{\nabla\varphi}_{-E}\cdot E\, dV\right]$$

$$= \frac{\epsilon_0}{2}\left[\int_{\partial\Omega}\varphi E\cdot n\, dA + \int_\Omega |E|^2\, dV\right]$$

This is the so-call <u>Integration By Parts</u>

$$\underbrace{\int_\Omega \varphi(\nabla\cdot E)\, dV}_{u\,dv} = \underbrace{\int_{\partial\Omega}\varphi E\cdot n\, dA}_{uv} - \underbrace{\int_\Omega \nabla\varphi\cdot E\, dV}_{du\ v}$$

They then make an arg that the charge is on a bdd set ($\mathrm{supp}(\rho)$ is bdd). We can increase the size of Ω so long as it still contains all our charge. As Ω is increased, the term $\int_{\partial\Omega}$ decreases as $1/r$ while $\int_\Omega E^2$ could only incr. Let Ω grow to all of $\mathbb{R}^3$

and $W = \frac{\epsilon_0}{2}\int_{\mathbb{R}^3} E^2\, dV$

▷ Let's solve the solid ball problem again using this $\vec{E}_{in} = \frac{kQ}{R^3} r\hat{e}_r$

$\vec{E}_{out} = \frac{kQ}{r^2}\hat{e}_r$

$$W = \frac{\epsilon_0}{2}\int_{0}^{R} E_{in}^2\, dV + \int_{\mathbb{R}^3 - B(0,R)} E_{out}^2\, dV = \frac{\epsilon_0}{2}\left[\left(\frac{kQ}{R^3}\right)^2 \int_0^R\int_0^\pi\int_0^{2\pi} r^2(r^2\sin\phi\, d\theta\, d\phi\, dr) + k^2Q^2\iiint \frac{1}{r^4} r^2\sin\phi\right]$$

really $B(0,R)$

$$= \frac{\epsilon_0}{2}\left[\frac{k^2Q^2}{R^6}4\pi\int_0^R r^4\, dr + k^2Q^2 4\pi\int_R^\infty \frac{1}{r^2}\, dr\right]$$

$$= \frac{\epsilon_0}{2}\left[\frac{k^2Q^2 4\pi}{\cancel{R^6}}\frac{1}{5}\cancel{R^5} + k^2Q^2 4\pi\left(-\frac{1}{r}\right)\Bigg|_R^\infty\right] = \frac{\epsilon_0}{2}k^2Q^2 4\pi\left[\frac{1}{5R} + \frac{1}{R}\right] = \frac{\epsilon_0}{2}\frac{k}{4\pi\epsilon_0}\frac{Q^2 4\pi}{R}\frac{6}{5}$$

$\left(-0 + \frac{1}{R}\right)$ $\quad \frac{6}{5}\frac{1}{R}$

$$= \frac{3}{5}\frac{kQ^2}{R}$$ Same as before □

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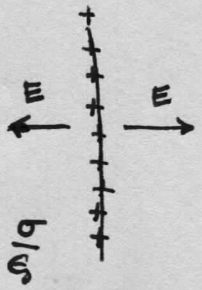
This is starting (II) which I listed on sheet (6)

The goal now is to establish $\frac{\partial \varphi}{\partial \hat{n}} = \frac{\sigma}{\epsilon_0}$ and later this will give us Neumann boundary conds for PDE

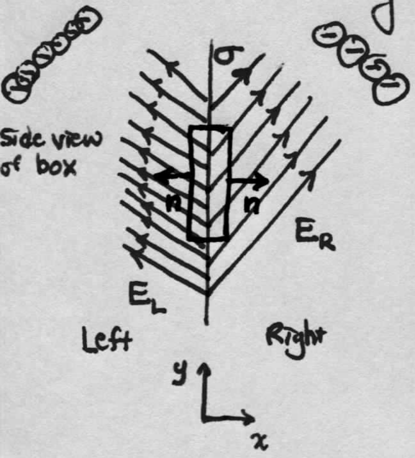
Consider again a charged sheet (ch 5 sheet (3))

E field is $\perp$ on both sides if no other charges nearby



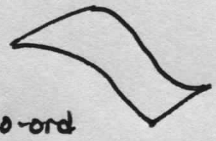
Gauss box arg $\Rightarrow \vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n}$ Crossing the sheet gives a jump discontinuity $\Delta E = \left(\frac{1}{2}\frac{\sigma}{\epsilon_0} - -\frac{1}{2}\frac{\sigma}{\epsilon_0}\right) = \frac{\sigma}{\epsilon_0}$

▷ But now, following Griffith p. 88-90 and Purcell p. 92-93



Due to external charges, the E lines are different on left and right sides (and not $\perp$ to sheet) so $\vec{E}_L$ and $\vec{E}_R$

Also the sheet is not nec a plane, but we are just looking at it up close, so locally is appears flat



We separate $\vec{E}$ into $E^{\parallel}$ and $E^{\perp}$

$E^{(y)}$ and $E^{(x)}$ with our co-ord sys.

The box is very thin, so an arb small amount of flux goes thru the sides $\perp$ to sheet. We need only consider flat faces S_R and S_L

Gauss: $\int_S \vec{E}\cdot\hat{n}\,dA = \frac{1}{\epsilon_0} Q_{inside} = \frac{1}{\epsilon_0}\sigma A$

$$\int_{S_L} \vec{E}_L\cdot(-\hat{e}_x)\,dA + \int_{S_R} \vec{E}_R\cdot(\hat{e}_x)\,dA = \frac{\sigma A}{\epsilon_0}$$

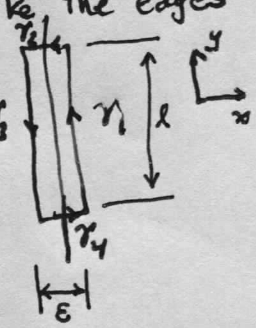
$$(E_L^x + E_L^y)\cdot(-\hat{e}_x)A + (E_R^x + E_R^y)\cdot\hat{e}_x A \Rightarrow (-E_L^x + E_R^x)\cancel{A} = \frac{\sigma}{\epsilon_0}\cancel{A}$$

$\Rightarrow$ $\boxed{\Delta E^x = \frac{\sigma}{\epsilon_0} \text{ always this jump discontinuity in normal dir when crossing a charged surf}}$

▷ Now show $E^{\parallel}$ is continuous (same value as we approach surf from either side)

we know $\oint_\gamma \vec{E}\cdot d\vec{s} = 0$ so let γ be a thin rectangle, it looks just like the edges of the box above

side view



$$0 = \int_{\gamma_1}(E_R^x + E_R^y)\cdot\hat{e}_y\,dy + \int_{\frac{1}{2}\gamma_2} -E_R^x\,dx + \int_{\frac{1}{2}\gamma_2} E_L^x\,dx + \int_{\gamma_3} E_L^y\cdot(-\hat{e}_y)\,dy + \int_{\frac{1}{2}\gamma_4} -E_L^x + \int_{\frac{1}{2}\gamma_4} E_R^x$$

$$= \int_{\gamma_1} E_R^y\,dy - \int_{\gamma_3} E_L^y\,dy = \varepsilon\,(\text{Top} + \text{bot})$$

$\boxed{\varepsilon \text{ is } \perp \text{ dist from sheet}}$

$= E_R^y(\varepsilon)l - E_L^y(-\varepsilon) = \varepsilon(\dots);\ \lim_{\varepsilon\to 0} \Rightarrow \boxed{E_R^y(0^+) = E_L^y(0^-)}$

Combine E^x and E^y:

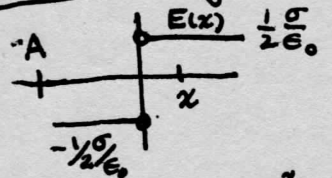
$$\boxed{\vec{E}_R(0^+) - \vec{E}_L(0^-) = \frac{\sigma}{\epsilon_0}\hat{n}}$$

eq (2.33)

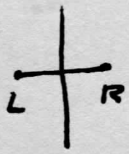
This is also called 'Coulomb's Law' Jeans p. 45

So $\vec{E}$ is discont crossing the surf. But potential fcn φ is cont:

sketch of arg:



$$\varphi_A(x) = -\int_{-A}^{x} \vec{E}\cdot ds = \int_{-A}^{x} E(x)\,dx$$ ord fcn of 1 variable



want: $\lim_{x \nearrow 0} \varphi_A(x) = \lim_{x \searrow 0} \varphi_A(x)$

"integration is a smoothing operation"

$$\lim_{x \nearrow 0} \int_{-A}^{x} E(x)\,dx = \int_{-A}^{0} -\frac{\sigma}{2\epsilon_0}\,dx = -\frac{\sigma}{2\epsilon_0}A$$

whereas $\lim_{x \searrow 0} \int_{-A}^{x} E(x)\,dx = -\frac{\sigma}{2\epsilon_0}A + \cancel{x\left(\frac{\sigma}{2\epsilon_0}\right)}^{\,0}$ ✓

▷ But $\nabla\varphi$ does inherit the discontinuity:

$$\vec{E}_R(0^+) - \vec{E}_L(0^-) = \frac{\sigma}{\epsilon_0}\hat{n}$$

$$-\nabla\varphi(0^+) - {-\nabla\varphi(0^-)} =$$

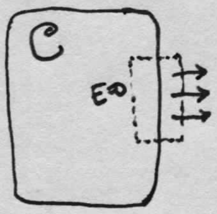
$$\nabla\varphi(0^+) - \nabla\varphi(0^-) = -\frac{\sigma}{\epsilon_0}\hat{n}$$ dot prod both sides with $\hat{n}$

normal deriv

$$\nabla\varphi_{0^+}\cdot n - \nabla\varphi_{0^-}\cdot n = -\frac{\sigma}{\epsilon_0}$$

$$\delta\varphi_{0^+}(n) - \delta\varphi_{0^-}(n) = -\sigma/\epsilon_0 \qquad (2.36)$$

▷ Griffith p.102 **§2.5.3 Surf Charges and Force on a Conductor**



$$\vec{E}_R(0^+) - \vec{E}_L(0^-) = \frac{\sigma}{\epsilon_0}\hat{n}$$ but E_L is 0 in Conductor

$$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{n}$$ Then Griffith rewrites (2.36) as (justification?)

$$\boxed{\frac{\partial\varphi}{\partial n} = \frac{-\sigma}{\epsilon_0}} \Rightarrow \boxed{\sigma = -\epsilon_0 \frac{\partial\varphi}{\partial n} \quad (2.49)}$$

Feynman ch 5-10
Griffith ch 2.5.2 p. 98-101

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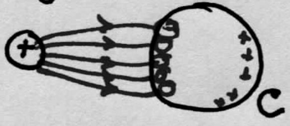
(III) The field inside a Cavity in a conductor. Electrostatic shielding

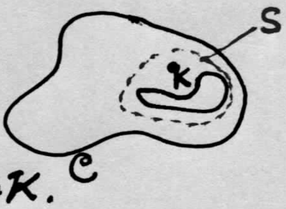
Remark: An external charge will attract a neutral conductor – to cancel the field inside C e^- flow to nearest side and this results in a force of attraction on the macroscopic bodies.



▷ $E = 0$ inside an empty cavity K (just like there was no cavity there)

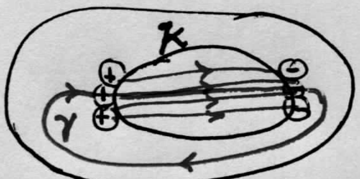
(i) By cons of energy $E = 0$ inside the metal of C (steady state)

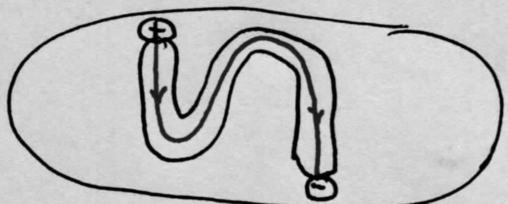
(ii) enclose K by a Gaussian surf S wholly contained in C
There is no flux thru S ⇒ no net charge inside S



(iii) No charge separation on the walls of K. No E flow lines in K.

Suppose charge separation did exist:





Case 1: Easy path from pos to neg
Integrate along closed loop γ following E inside K

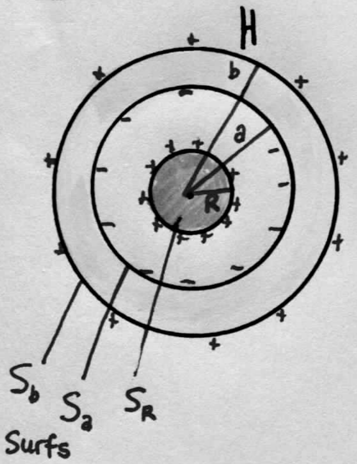
$$\oint_\gamma E\cdot ds \overset{!}{=} 0 \quad \text{but} \quad \int_\gamma E\cdot ds = \underbrace{\underbrace{\int E\cdot ds}_{\text{in } K}}_{\text{pos}} + \underbrace{\int_{\text{in } C} E\cdot ds}_{0} > 0 \Rightarrow\Leftarrow$$

Case 2 Ridiculous path
E lines can't touch C nearby to pos charge, or else they have induced a neg charge there and we are back to case 1. So we take the ridiculous path and still case 1 arg applies.

Now we have a charge in cavity

Griffiths p.101 #2.35

Metal sphere $S_{(0,R)}$ carries charge q. It is surrounded by concentric spherical thick shell H with no charge.



(a) Find charge density on all surfs

Surf S_R has unif charge density $\sigma_R = \frac{q}{4\pi R^2}$
Its E field is identical to a pt charge q at origin

$$E_{S_R} = \begin{cases} 0 & 0<r<R \text{ (field is 0 inside unif charged sphere)} \\ \frac{kq}{r^2}\hat{e}_r & r>R \end{cases}$$

To cancel E_{S_R} inside H (metal of H) the inner surf S_a must have a unif charge $\sigma_a = \frac{-q}{4\pi a^2}$

This makes
$$E_{S_a} = \begin{cases} 0 & 0<r<a \\ \frac{-kq}{r^2}\hat{e}_r & r>a \end{cases}$$

Thus
$$\vec{E}_{S_a} + \vec{E}_{S_R} = \begin{cases} 0 & 0<r<R \\ \frac{kq}{r^2}\hat{e}_r & R<r<a \\ 0 & a<r \end{cases}$$

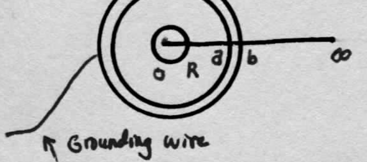
Finally S_b has a unif surf charge $\sigma_b = \frac{+q}{4\pi b^2}$ It is all the unpaired pos charge left over when S_a was neg charged.

$$E_{S_b} = \begin{cases} 0 & 0<r<b \\ \frac{kq}{r^2}\hat{e}_r & b<r \end{cases}$$

Thus the outside world does feel the charge in the cavity. We could surround H by a Gaussian surf to see flux.

cont'd →

(b) What is the value of the potential fcn φ at the origin?



$$\varphi(0) = -\int_{\infty}^{0} \vec{E}\cdot d\vec{s} = -\int_{\infty}^{b} \frac{kq}{r^2}\hat{e}_r\cdot\hat{e}_r\,dr + -\int_{b}^{a} 0 + -\int_{a}^{R} \frac{kq}{r^2}\,dr + -\int_{R}^{0} 0$$

$$\uparrow\ \vec{E}_{S_a} + \vec{E}_{S_R}$$

$$= -kq\left(\left[\frac{-1}{r}\right]_{\infty}^{b} - \left[\frac{1}{r}\right]_{a}^{R}\right) = +kq\left[\frac{1}{b} + \frac{1}{R} - \frac{1}{a}\right]$$

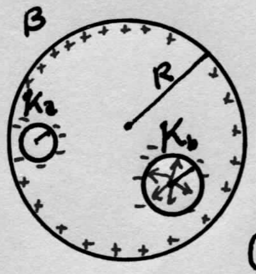
(c) If the outer surf S_b is now grounded, how do (a) and (b) change?

Pos charge flows off outer surf. Other surfs unaffected.

$\sigma_b = 0 \qquad \vec{E}_{S_b} = \vec{0}$ outside world is shielded from charge in cavity.

$$\varphi(0) = -\int_{a}^{R} \frac{kq}{r^2}\,dr = kq\left(\frac{1}{R} - \frac{1}{a}\right) \quad \square$$

(2.36) A solid spherical ball B has 2 arbitrarily placed spherical cavities K_a and K_B. Charges are in the center of each cavity. K_a has radius r_a and charge inside q_a; likewise K_b r_b q_b



(a) Surf charge densities $\sigma_a = \dfrac{-q_a}{4\pi r_a^2} \qquad \sigma_b = \dfrac{-q_b}{4\pi r_b^2} \qquad \sigma_R = \dfrac{(q_a + q_b)}{4\pi R^2}$

If say K_a was close to surface, it doesn't matter as long as there is a nonzero thickness of metal in between. $E = 0$ in the interior of metal, so surf charge on B doesn't know about cav K_a.

(b) Outside B, the sphere B looks just like a pt. charge at origin

$$\vec{E}(r) = \frac{k(q_a + q_b)}{r^2}\hat{e}_r \quad \text{for } R < r$$

(c) What is the field inside K_a? Just a pt charge at the center.

Let r_a be radial vector with origin at center of K_a

inside K_a $\quad \vec{E}_a = \dfrac{k\,q_a}{r_a^2}\hat{e}_{r_a}$

(d) What is the force on charge q_a? $F = q_a E$ where E is the total field from σ_a

Since q_a is in the center, all forces are balanced and $F = 0$

(Even if it were not in the center, for a unif charged sphere $S_a = \partial K_a$ there is no net force inside we know from gravitation that)

(e) Now let a 3rd charge q_c be brought near B from outside. What changes?

K_a and K_b are shielded so no change to σ_a, E_a and no force (same for b)

The surf density σ_R would change and so would $E_{outside}$ (but we can't say exactly how with the techniques we have now)

Purcell p.96

<u>Remark</u>: Electrostatic shielding still holds in practice for E fields that don't vary 'too fast'. We could take this to mean an order of magnitude longer than it would take light to travel the length of the enclosure. That is still quite fast!

This perhaps explains why a metal car will protect you from electricity going thru you in a lightening strike.