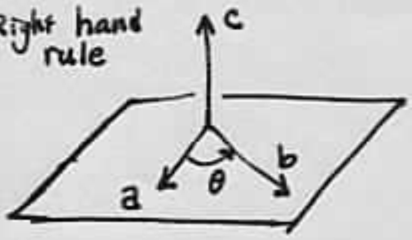
Cross product (vector product)

Right hand rule



Cross product only exists in $\mathbb{R}^3$

If $\vec{a} \times \vec{b} = \vec{c}$ then $\vec{a} \perp \vec{c}$ and $\vec{b} \perp \vec{c}$

$\|\vec{a} \times \vec{b}\| = \|\vec{a}\| \, \|\vec{b}\| \, |\sin\theta|$ $\quad$ $\sin\theta$ pos for $\theta \in [0, \pi)$

$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$ $\quad$ anti-commutative

$$\vec{a} \times \vec{b} = \det \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ -(a_1 b_3 - b_1 a_3) \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

we can also represent $\vec{a} \times (\cdot)$ as a matrix operator:

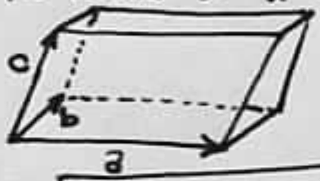
$$a \times (b) = \Lambda_a(b) = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

anti-symm

Triple Scalar product $\quad \vec{a} \cdot (\vec{b} \times \vec{c}) = \det \begin{bmatrix} - a - \\ - b - \\ - c - \end{bmatrix}$

so from this we can see how to permute a, b, c

This is also the volume of a parallelipiped



MBT p.39

Triple Vector product $\quad \boxed{\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}\,(a \cdot c) - \vec{c}\,(a \cdot b)}$ $\quad$ 'baccab rule'

$(a \times b) \times c = \vec{b}\,(a \cdot c) - \vec{a}\,(b \cdot c)$

CROSS PROD NOT ASSOCIATIVE!

Jacobi identity: $(a \times b) \times c + (b \times c) \times a + (c \times a) \times b = 0$

circular perm $a \, b \, c$

$$(\vec{u} \times \vec{v}) \cdot (\vec{p} \times \vec{q}) = (u \cdot p)(v \cdot q) - (u \cdot q)(v \cdot p) = \det \begin{bmatrix} u \cdot p & u \cdot q \\ v \cdot p & v \cdot q \end{bmatrix}$$

Remark about oriented bases in $\mathbb{R}^3$:

Given 2 LI vectors $\vec{a}, \vec{b} \in \mathbb{R}^3$, how can we make a pos oriented basis for $\mathbb{R}^3$?

Just take $\{a, b, a \times b\}$ This is pos because $\det \begin{bmatrix} - a - \\ - b - \\ a \times b \end{bmatrix} = (-1)^2 \det \begin{bmatrix} a \times b \\ - a - \\ - b - \end{bmatrix} = (a \times b) \cdot (a \times b) = \|a \times b\|^2 > 0$